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Predictive functional control based on an adaptive fuzzy model of a hybrid semi-batch reactor

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ABSTRACT

In this paper a new approach to the control of a nonlinear, time-varying process is proposed. It is based on a recursive version of the fuzzy identification method and predictive functional control. First, the recursive fuzzy identification method is derived, after which it is used in connection with fuzzy predictive functional control to construct an adaptive fuzzy predictive functional controller. The adaptive FPFC is then tested on a nonlinear, time-varying, semi-batch reactor process and compared with the standard FPFC, which uses non-adaptive fuzzy model. The simulation results are promising; they indicate that the control of time-varying, nonlinear processes with the FPFC can be improved with the use of an adaptive fuzzy model. An improvement in reference tracking and disturbance rejection can be observed, but the main advantage is the reduced number of switchings between hot and cold water. This is an important improvement in the case of real applications.

1. Introduction

In chemistry, pharmacy and biotechnology the batch reactor is one of the most important parts of the process technology. The control of a batch reactor is essentially a problem of temperature control, which is difficult to overcome. The difficulties arise from the mixed continuous and discrete nature of the process behavior, the various uses of these reactors, the drastic changes in the set-point during the operation, and the different modes of operation. Large numbers of reactors are semi-batch reactors in which the initial mixture of material is placed. The mixture is then heated to the desired temperature and additional reactant is added to the vessel during the procedure. The volume in the reactor varies, tending to increase with time, as does the heat-transfer surface of the reactor. From the dynamics point of view the reactor represents a time-varying process with unknown parameters. The optimal operation of the semi-batch reactor is to follow the reference trajectory, which is defined by a technological recipe, as precisely as possible and without overshoot (Škrjanc, 2008).

In the literature a number of papers have been published that discuss the control of semi-batch reactors and various types of controllers were studied. The control schemes up to 1986 are given in Juba and Hamer (1986). Since then many different concepts of semi-batch reactor control were developed. The most promising of these were the concepts of adaptive control (Chen, Bastin, & Van Breusegam, 1995; Louleh, Cabassud, & Le Lann, 1999), optimal control (Chang & Hseih, 1991; Cuthrell & Biegler, 1989; Luus & Okongwu, 1999) and especially model predictive control schemes, which are the most frequently used (Clarke, 1994; Clarke, Mohtadi, & Tuffs, 1987; De Vries & Verbruggen, 1994; Foss, Johansen, & Sorensen, 1995; Henson & Seborg, 1995; Lakshmanan & Arkun, 1999; Loeblin, Perkin, Srinivasan, & Bonvin, 1999). Model predictive control was very successful in solving many industrial control problems (Bequette, 1991; Clarke & Mohtadi, 1989; Cutler & Ramaker, 1980). Two of the most frequently used predictive schemes in practice are the predictive functional control scheme (Richalet, 1993) and the fuzzy predictive functional control scheme (Škrjanc & Matko, 2000), which was also used as a base algorithm in this study. In Karer, Škrjanc, and Zupančič (2008) and in Škrjanc (2007) studies of adaptive predictive control for semi-batch reactor can also be found.

Predictive control based on the fuzzy model (FPFC) proved to be very convenient for nonlinear processes (Lepetič, Škrjanc, Chiacchiarini, & Matko, 2003; Škrjanc & Matko, 2001). The method is based on the fact that a smooth nonlinear process can be represented by a set of linear models in the form of a Takagi–Sugeno fuzzy model (Takagi & Sugeno, 1985; Wang, 1994).

When dealing with nonlinear, time-varying processes, like the example presented in this paper (semi-batch reactor process), the fuzzy model should update (adapt) itself to new process conditions in order to maintain the quality of the control. Therefore, an on-line fuzzy identification method should be used to properly adapt the membership functions and parameters of the Takagi–Sugeno model.

There are many methods that deal with on-line fuzzy model identification, for example, the evolving fuzzy neural network (EFuNN) (Kasabov, 2001), the dynamic evolving neural-fuzzy

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inference system algorithm (DENFIS) (Kasabov & Song, 2002), the evolving Takagi–Sugeno model (eTS) (Angelov & Filev, 2004), the self-organizing fuzzy neural network (SOFNN) (Qiao & Wang, 2008), the generalized adaptive neuro-fuzzy inference system (GANFIS) (Azeem, Hanmandlu, & Ahmad, 2003) and others (Angelov, 1995; Deng & Kasabov, 1991; Juang & Lin, 1998; Lin, 1995; Lin, Lin, & Shen, 2001; Patt, 1991; Rong, Sundararajan, Huang, & Saratchandran, 2006; Tzafestas & Zikidis, 2001; Wu & Er, 2000; Wu, Er, & Gao, 2001). The fuzzy model identification is composed of the identification of the membership functions' parameters' and the identification of the sub-models' parameters. For the sub-models' parameters identification a recursive least-square method is used. The identification of the membership functions' parameters is made using clustering methods. This is the part where the above-mentioned methods differ. For example, DENFIS uses the evolving clustering method (ECM) (Kasabov & Song, 2002), eTS uses the recursive version of subtractive clustering (Fritzke, 2004) and the SOFNN uses the rival penalized algorithm (RLPC) (Xu, Krzyzak, & Oja, 1993).

In this paper a version of the on-line fuzzy-identification method was derived. The proposed method is a recursive version of the established fuzzy *c*-means identification method. The identification method is similar to Angelov and Filev's on-line clustering (Angelov & Filev, 2004). The difference is that their method is based on the subtractive clustering off-line method (Chiu, 1994), whereas the proposed method is based on the fuzzy *c*-means off-line clustering method (Bezdek, 1981). The positions of the centers in the proposed method depend on a weighted mean of the data belonging to the *i*-th cluster, whereas with Angelov's method the centers can only be the data samples with the largest potential. Also, the width of the membership function in the proposed method depends on the fuzzy variance and changes depending on the data pattern, whereas with Angelov's method the width is fixed and predefined. Both methods use the Gaussian membership functions. The local linear sub-models are, in both cases, updated with recursive least squares.

The developed recursive fuzzy identification algorithm is then used in connection with the recursive least squares to obtain the membership functions' parameters and the parameters of the fuzzy model. The adaptive fuzzy model is implemented in the predictive functional control algorithm to construct the adaptive FPFC algorithm. Together they form the indirect adaptive fuzzy controller. The idea of recursive fuzzy *c*-means clustering can also be used as a self-tuning method for the proposed controller.

Over the past decade a lot of work has been done in the field of adaptive fuzzy logic control. In general the schemes can be divided into direct and indirect adaptive schemes. The direct schemes approximate the ideal controller with fuzzy logic (Boulkroune, Tadjine, Saad, & Farza, 2010; Chen, Tan, Han, & Wang, 1997; Kim, Kim, & Park, 1996; Labiod & Guerra, 2007; Lee, Lee, & Kang, 1996; Salehi & Shahrokhi, 2008; Tong, He, Li, & Zang, 2010; Tong, Li, & Shi, 2009; Wang, 1993). The indirect schemes use a fuzzy system to approximate the plant dynamics (Chan, Rad, & Wang, 2001; Golea, Golea, & Benmahammed, 2003; Qi & Brdys, 2008; Wang, Ge, & Lee, 2000; Wang, Rad, & Chan, 2001). The adaptive fuzzy schemes usually apply only the adaptation of the sub-models' parameters (consequent parameters) and not also the width and centers of the membership functions (premise parameters) as in the proposed method. The membership functions are usually predefined and kept constant, only the sub-models' parameters of the fuzzy model are adapted. Very few versions apply the adaptation of both: In Singh (1998) a direct approach that uses triangular membership functions and the adaptation of its centers is proposed. In Phan and Gale (2008) a direct algorithm that also uses triangular membership functions is proposed. The algorithm adds or replaces the membership functions depending on the error threshold. A direct approach is also proposed in Rojas et al. (2006). The approach uses triangular functions and tries to find the membership function configuration that distributes a certain performance criterion homogeneously through out the operating regions. The approach presented uses the squared error as a performance criterion. The indirect adaptive approach is proposed in Qi and Brdys (2008). The approach uses Gaussian membership functions and adapts both the width and the centers. The adaptation of the centers and the widths of the membership functions is done using a gradient decent algorithm and the adaptation of sub-models' parameters is done using recursive least squares. The approach is similar to the one described here. The difference is that the proposed algorithm uses the recursive fuzzy *c*-means method to adapt the width and the centers of the membership functions.

The adaptive FPFC developed in this paper is compared to the non-adaptive FPFC. The results show that the adaptive controller can cope with the change of the system dynamics and can maintain the quality of the control. The advantage of the adaptive version is that the model adapts itself to the new process dynamics. The fuzzy model can therefore also be used not only for the FPFC control algorithm but also for parallel fault detection.

The paper is organized in the following fashion. First, the recursive fuzzy *c*-means clustering algorithm is derived. Then the idea of fuzzy predictive control is explained and the control law is given. Then follows the description of the semi-batch reactor process. Finally, the simulation results are presented and the adaptive fuzzy predictive functional control is compared to fuzzy predictive control.

2. Fuzzy *c*-means and recursive fuzzy *c*-means clustering

The fuzzy model represents nonlinear mapping between the input and output variables. Dynamic systems are usually modelled by feeding back the delayed input and output signals. The common nonlinear model structure is nonlinear autoregressive with an exogenous (NARX) input model:

$$\hat{y}(k+1) = F(y(k),\ldots,y(k-n+1),u(k),\ldots,u(k-m+1)) \quad (1)$$

where the $y(k),\ldots,y(k-n+1)$ and $u(k),\ldots,u(k-m+1)$ denote the delayed model input and output. The fuzzy model approximates the function F. The model can be obtained by means of modelling or using identification methods. Fuzzy models utilize the idea of linearization in a fuzzily defined region of the state space. The nonlinear model is decomposed into a multi-model structure consisting of linear models (Johanson & Murray-Smith, 1981). Some of the most useful and wide spread fuzzy models are the Takagi–Sugeno fuzzy models.

The model identification of the Takagi–Sugeno model consists of the identification of the clusters (fuzzy regions) and the identification of the linear sub-models' parameters that are valid for a certain cluster. The identification of the clusters can be made using clustering algorithms such as fuzzy *c*-means and the parameter estimation can be made using a least-squares method.

2.1. The fuzzy c-means clustering

The *c*-means clustering algorithm clusters the data into a predefined number of clusters. An assumption that each observation consists of m samples is made. They are grouped into an m-dimensional vector $x(k)^T=[x_1(k),\ldots,x_m(k)]$, $x(k)\in\mathbb{R}^m$, where $x_m(k)$ stands for the m-th measurement at the time instant k. A set of n observations is then denoted as $X=\{x(k)|k=1,2,\ldots,n\}$, $X\in\mathbb{R}^{n\times m}$.

The main objective of the clustering is to partition the data set X into c subsets, which are called clusters. The data matrix X is

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given as follows:

$$X=\begin{bmatrix} x_1(1) & x_2(1) & \dots & x_m(1) \\ x_1(2) & x_2(2) & \dots & x_m(2) \\ \vdots & \vdots & \vdots & \vdots \\ x_1(n) & x_2(n) & \dots & x_m(n) \end{bmatrix} \tag{2}$$

The data vector at the time instant k is defined as (the rows of the matrix X) $x(k)^T=[x_1(k),\dots,x_m(k)], x(k)\in\mathbb{R}^m$. The fuzzy partition of the set X is a family of fuzzy subsets $\{A_i|1\le i\le c\}$. These fuzzy subsets are defined by their membership functions, which are implicitly defined in the fuzzy partition matrix $U=[\mu_i(k)]\in\mathbb{R}^{c\times n}$. The i-th row of the matrix U contains the values of the membership function of the i-th fuzzy subset A_i of the data matrix X. The partition matrix satisfies the following conditions: the membership degrees are real numbers from the interval $\mu_i(k)\in[0,1], 1\le i\le c, 1\le k\le n$; the total membership of each of the samples in all the clusters equals one $\sum_{i=1}^{c}\mu_i(k)=1, 1\le k\le n$; and none of the fuzzy clusters is empty nor do any contain all the data $0<\sum_{k=1}^{n}\mu_i(k)<n$, $1\le i\le c$. This means that the fuzzy partition matrix U belongs to the fuzzy partition set, which is defined as

$$M=\left\{U\in\mathbb{R}^{c\times n}|\mu_i(k)\in[0,1]\ \forall i,k;\sum_{i=1}^{c}\mu_i(k)=1,\forall k;0<\sum_{k=1}^{n}\mu_i(k)<n\ \forall i\right\} \tag{3}$$

The c-means algorithm for clustering in n dimensions produces c-mean vectors that represent c classes of data. The algorithm relies on a distortion measure $d(x(k),v_i)$ between the points in data space $x(k),v_i\in\mathbb{R}^m$, where $x(k)$ denotes a certain point in that space, an observation, and v_i stands for the centroid. A variety of different norms can be used to define the distortion measure, such as L_1, L_2 and L_∞, or any other that is specific to the problem. The algorithm is based on a minimization of the fuzzy c-means objective function, which is introduced as the weighted-criterion function

$$J(X,U,V)=\sum_{i=1}^{c}\sum_{k=1}^{n}(\mu_i(k))^{\eta}d^2(x(k),v_i) \tag{4}$$

subject to the constraints

$$\sum_{i=1}^{c}\mu_i(k)=1\quad\forall k \tag{5}$$

where V is a matrix of cluster centroid vectors v_i, $V=[v_1,\dots v_c]^T$, and the overlapping factor or the fuzziness parameter η that influences the fuzziness of the resulting partition is denoted as η; from the hard ($\eta=1$) to the partition that is completely fuzzy ($\eta\to\infty$). In the proposed approach the standard value $\eta=2$ is used. In the case of the classical c-means clustering algorithm the distortion measure is defined as L_2 norm.

The problem of finding the fuzzy clusters in the data set X is now solved as a constrained optimization problem using Lagrange multipliers, which consider the minimization of the function in Eq. (4) over the domain X, and taking into account the constraints in Eq. (5). The minimum is obtained via the Lagrange multipliers method and is given as follows:

$$\mu_i(k)=\left(d_{ik}^2\sum_{j=1}^{c}\left(\frac{1}{d_{jk}^2}\right)^{1/(\eta-1)}\right)^{-1} \tag{6}$$

where d_{ik} defines the Euclidian distance (L_2-norm) between the observation $x(k)$ and the cluster centroid v_i as follows:

$$d_{ik}^2=(x(k)-v_i)^T(x(k)-v_i),\quad 1\le i\le c,\ 1\le k\le n \tag{7}$$

The cluster centroid v_i is defined as the weighted mean of the data belonging to the *i-th* cluster, where the weights are the membership degrees and are given as follows:

$$v_i(k)=\frac{\sum_{k=1}^{n}\mu_i^{\eta}(k)x(k)}{\sum_{k=1}^{n}\mu_i^{\eta}(k)} \tag{8}$$

2.2. The recursive fuzzy c-means

When the behavior of the process that generates the observed data changes over time, the clustering should be done recursively to obtain the clusters that describe the current behavior. To develop the recursive fuzzy clustering algorithm, first the cluster centroid vector $v_i^T=[v_{i1},\dots,v_{i_m}]$ is defined, according to the current observation, i.e., the weighted mean of the data according to the current membership degrees. This introduces the notation $v_i(r)$, which means the cluster centroid at the time instant r that is obtained by weighting with the current membership degrees. Form Eq. (8) the cluster centroid in the next observation is derived

$$v_i(r+1)=\frac{\sum_{k=1}^{r}\mu_i^{\eta}(k)x(k)+\mu_i^{\eta}(r+1)x(r+1)}{\sum_{k=1}^{r}\mu_i^{\eta}(k)+\mu_i^{\eta}(r+1)} \tag{9}$$

where $\mu_i(k), k=1,\dots,r+1$ denotes the membership degree of the observation vector $x(k)^T=[x_1(k),\dots,x_m(k)]$, $k=1,\dots,r+1$ to the cluster i at the time instant k. Introducing the relation between the old cluster centroid and a new one is as follows:

$$v_i(r+1)=v_i(r)+\Delta v_i(r+1) \tag{10}$$

and taking into account Eq. (9) the following is obtained:

$$\Delta v_i(r+1)=\frac{\mu_i^{\eta}(r+1)(x(r+1)-v_i(r))}{\sum_{k=1}^{r}\mu_i^{\eta}(k)+\mu_i^{\eta}(r+1)} \tag{11}$$

The cluster centroid increment in Eq. (11) cannot be calculated in the present form because the denominator in Eq. (11) cannot be recursively calculated. The calculation of the membership degrees requires all past r observations. This is against the recursive approach. An approximate calculation of this term can be made by introducing the exponential weighting of the past membership degrees, which are calculated at each time instant. The weights of the past data are decreasing exponentially.

Let us denote the term in the denominator of Eq. (11) as $s_i(r+1)\in\mathbb{R}^c$. This is calculated as

$$s_i(r+1)=s_i(r)+\mu_i^{\eta}(r+1) \tag{12}$$

where $s_i(r)$ is defined as follows:

$$s_i(r)=\sum_{k=1}^{r}\mu_i^{\eta}(k) \tag{13}$$

Introducing the forgetting factor, Eq. (12) can be rewritten as

$$s_i(r+1)=\gamma_v s_i(r)+\mu_i^{\eta}(r+1) \tag{14}$$

The parameter γ_v, $(0\le\gamma_v\le 1)$ denotes the forgetting factor of a past observation, i.e., the forgetting factor of the past membership degrees. The $\Delta v_i(r+1)$ can now be written as

$$\Delta v_i(r+1)=\frac{\mu_i^{\eta}(r+1)(x(r+1)-v_i(r))}{s_i(r+1)} \tag{15}$$

The current membership degree $\mu_i(r+1)$ is next defined as follows:

$$\mu_i(r+1)=\left(d_{i,r+1}^2\sum_{j=1}^{c}\left(\frac{1}{d_{j,r+1}^2}\right)^{1/(\eta-1)}\right)^{-1} \tag{16}$$

where $d_{i,r+1}^2$ defines the quadratic distance from the cluster centroid as follows:

$$d_{i,r+1}^2=(x(r+1)-v_i(r))^T(x(r+1)-v_i(r)),\quad 1\le i\le c \tag{17}$$

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Around each cluster centroid the distribution of the data can be described using the fuzzy covariance matrix, $F_i(r) \in \mathbb{R}^{m\times m}$, $i=1,\ldots,c$ and r stands for the number of samples used to calculate the matrix. The fuzzy covariance matrix is defined by the weighted covariance matrix as follows:

$$F_i(r)=\frac{\sum_{k=1}^{r}\mu_i^{\eta}(k)(x(k)-v_i^r)(x(k)-v_i^r)^T}{\sum_{k=1}^{r}\mu_i^{\eta}(k)} \tag{18}$$

where v_i^r stands for the centroid vector of the i-th cluster calculated for the set of r samples. The fuzzy covariance matrix for the next sample $(r+1)$ can be expressed as follows:

$$F_i(r+1)=\frac{\sum_{k=1}^{r}\mu_i^{\eta}(k)(x(k)-v_i^{r+1})(x(k)-v_i^{r+1})^T}{\sum_{k=1}^{r}\mu_i^{\eta}(k)+\mu_i^{\eta}(r+1)} + \frac{\mu_i^{\eta}(r+1)(x(r+1)-v_i^{r+1})(x(r+1)-v_i^{r+1})^T}{\sum_{k=1}^{r}\mu_i^{\eta}(k)+\mu_i^{\eta}(r+1)}, \tag{19}$$

where v_i^{r+1} stands for the centroid vector of the i-th cluster calculated for the set of $r+1$ samples. Taking into account Eq. (18), introducing it into Eq. (19), and using Eqs. (12) and (13), the following approximate recursive expression for the fuzzy clustering matrix is obtained

$$F_i(r+1)=\gamma_c\frac{s_i(r)}{s_i(r+1)}F_i(r)+\frac{\mu_i^{\eta}(r+1)}{s_i(r+1)}\cdot(x(r+1)-v_i(r+1)) \cdot(x(r+1)-v_i(r+1))^T \tag{20}$$

2.3. The identification of the sub-models

The centers of the fuzzy clusters and their distribution are used to define the new membership functions' distribution, and using the recursive least-squares method the fuzzy model is obtained. Using the projection of the cluster onto the independent variables, the input membership functions are obtained. The first $m-1$ measured variables in the data vector $x(k)$ represent the input variables and the last m-th variable represents the output. In the proposed method the clusters are approximated by the Gaussian membership functions, with the center v_i and the variance $\sigma_i^2=\eta_m^2\sum_{j=1}^{m}f_{i,j}$, where using η_m the overlapping between the membership functions is defined and $f_{i,j}$ stand for the diagonal elements of the matrix F_i. The membership function of the i-th cluster and the j-th component of $x(k)$ is therefore defined as

$$\mu_i(x_j(k))=e^{-((x_j(k)-v_{ij}(k))(x_j(k)-v_{ij}(k))^T)/2\sigma_i^2(k)},\quad i=1,\ldots,c,\ j=1,\ldots,m-1 \tag{21}$$

Each of the $m-1$ input variables defines the input subspace. The whole input hyperspace is defined as a Cartesian product of these subspaces. This implies the definition of the membership degree in each subspace as the product of the membership degrees, as follows:

$$\beta_i(k)=\prod_{j=1}^{m-1}\mu_i(x_j(k)) \tag{22}$$

The fuzzy recursive least-squares algorithm is then obtained as follows:

$$\begin{aligned}\psi_i^T(k+1)&=\beta_i(k)[1,x_1(k),x_2(k),\ldots,x_{m-1}(k)]\\ y_i(k)&=\beta_i(k)x_m(k)\\ P_i(k+1)&=\frac{1}{\lambda_r}\left(P(k)-\frac{P_i(k)\psi_i(k+1)\psi_i^T(k+1)P_i(k)}{\lambda_r+\psi_i^T(k+1)P_i(k)\psi_i(k+1)}\right)\\ \theta_i(k+1)&=\theta_i(k)+P_i(k)\psi_i(k+1)(y_i(k)-\psi_i^T(k+1)\theta_i(k))\end{aligned} \tag{23}$$

where λ_r stands for the exponential forgetting factor, which should be set between 0.98 and 1 to deal with time-varying processes, P_i stands for the covariance matrix, which is set to $P_i(0)=10^2-10^5I$, $I\in\mathbb{R}^{m\times m}$, and θ_i represents the parameters of the i-th local model and is written as follows:

$$\theta_i^T=[\theta_{i,0},\theta_{i,1},\theta_{i,2},\ldots,\theta_{i,m-1}] \tag{24}$$

Each local model contributes to the output of the model with the corresponding membership value. The whole set of fuzzy model parameters can be written in the matrix as follows:

$$\Theta=[\theta_1,\ \theta_2,\ \ldots,\ \theta_c] \tag{25}$$

3. Predictive functional control based on the fuzzy model

Model-based predictive control (MPC) is a control strategy based on the explicit use of a dynamic model of the process. The model is used to predict the future behavior of the process output signal over a certain finite horizon and to evaluate control actions to minimize a certain cost function. The predictive control law is in general obtained by minimizing of the following criterion:

$$J(u,k)=\sum_{j=N_1}^{N_2}(y_m(k+j)-y_r(k+j))^2+\lambda\sum_{j=1}^{N_u}u^2(k+j) \tag{26}$$

where $y_m(k+j)$, $y_r(k+j)$ and $u(k+j)$ stand for the j-step-ahead prediction of the process output signal, the reference trajectory and the control signal. N_1, N_2 and N_u are the minimum, maximum and control horizon. λ weights the relative importance of the control and output variables.

MPC stands for a collection of several different techniques all based on the same principles. Originally, the algorithms were developed for linear systems, but the basic idea of prediction has been extended to nonlinear systems (Clarke, 1994; Doyle, Ogunnaike, & Pearson, 1995). In the fuzzy predictive functional control the fundamental principles of predictive functional control are applied. These principles are very strong and easy to understand (Richalet, 1993; Richalet, Rault, Testud, & Papon, 1978).

The global, linear, first-order model of a smooth nonlinear process is described by the following difference equation with global linear parameters:

$$y_m(k+1)=\tilde{a}_m y_m(k)+\tilde{b}_m u(k)+\tilde{r}_m \tag{27}$$

When using the Takagi–Sugeno fuzzy model with first-order submodels, the model output y_m and the global parameters are calculated in the following fashion. The output of the i-th submodel is

$$y_{m,i}(k+1)=a_{m,i}y_{m,i}(k)+b_{m,i}u(k)+r_{m,i} \tag{28}$$

The global model output depends on the input data vector $x(k)^T=[y_m(k)\ u(k)]$. First, the membership degrees of the input data vector to the i-th cluster are calculated using Eqs. (21) and (22) (note that this vector contains only the input variables). Using the vector of membership degrees and the input data vector the output of the global model can be calculated:

$$y_m(k+1)=\Theta\beta\begin{bmatrix}1\\ x(k)\end{bmatrix} \tag{29}$$

where $\beta^T=[\beta_1,\ldots,\beta c]$ and

$$\Theta=\begin{bmatrix}r_{m,1}, & \ldots, & r_{m,c}\\ a_{m,1}, & \ldots, & a_{m,c}\\ b_{m,1}, & \ldots, & b_{m,c}\end{bmatrix} \tag{30}$$

The multiplication of the Θ and β matrices gives us the matrix under Eq. (31), from which the global parameters of the

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Takagi–Sugeno model are obtained.

$$\begin{bmatrix} \beta_1 r_{m,1} + \cdots + \beta_c r_{m,c} \\ \beta_1 a_{m,1} + \cdots + \beta_c a_{m,c} \\ \beta_1 b_{m,1} + \cdots + \beta_c b_{m,c} \end{bmatrix} \tag{31}$$

From the elements of the matrix under Eq. (31) the global parameters can be written as

$$\tilde{a}_m = \beta_1 a_{m,1} + \cdots + \beta_c a_{m,c} \tag{32}$$

$$\tilde{b}_m = \beta_1 b_{m,1} + \cdots + \beta_c b_{m,c} \tag{33}$$

$$\tilde{r}_m = \beta_1 r_{m,1} + \cdots + \beta_c r_{m,c} \tag{34}$$

Now the global linear model (Eq. (27)) is obtained. To derive the control law for fuzzy predictive functional control first the desired closed-loop behavior is specified. This is specified by the reference-trajectory given by the reference-model Eq. (35).

$$y_r(k+1) = a_r y_r(k) + (1-a_r) w(k) \tag{35}$$

This ensures that the reference output tracks a constant reference signal w. The control law is obtained from minimizing the criterion function Eq. (26). In fuzzy predictive functional control a single horizon is assumed $N_1 = N_2 = H$. This is called the coincidence horizon. For this horizon the predicted output value coincides with the reference trajectory. Taking into account the constant future control, the control law can be derived:

$$u(k) = \frac{(1-a_r^H)(w(k)-y_p(k))}{\dfrac{\tilde{b}_m}{1-\tilde{a}_m}(1-\tilde{a}_m^H)} + \frac{y_m(k)}{\dfrac{\tilde{b}_m}{1-\tilde{a}_m}} - \frac{\tilde{r}_m}{\tilde{b}_m} \tag{36}$$

A detailed derivation of the control law can be found in Škrjanc and Matko (2000).

4. The semi-batch reactor regulation

4.1. The process description

The method was tested on a model of a real semi-batch reactor stationed in a pharmaceutical company. A scheme of the semi-batch reactor is shown in Fig. 1. Dynamically, it is an example of a hybrid plant. The goal is to control the temperature of the ingredients stirred in the reactor's core, so that they synthesize optimally into the final product. In order to achieve this the temperature has to follow, as accurately as possible, the prescribed reference trajectory, with as low overshoot as possible.

The reactor's core (temperature T) is heated or cooled through the medium in the reactor's jacket (temperature T_j). These are also the measured outputs of the process. The medium in the jacket is a mixture of fresh input glycol, which enters the reactor through on/off valves, and reflux glycol. The temperature of the fresh, input glycol depends on whether the v_C is open or the v_H is open. If $v_C = 1$ then $v_H = 0$ and the input glycol temperature will be $T_{in} = T_C = -25\,^{\circ}\mathrm{C}$, and if $v_H = 1$ then $v_C = 0$ and the input glycol temperature will be $T_{in} = T_H = 130\,^{\circ}\mathrm{C}$. The ratio of fresh glycol to reflux glycol is controlled by a third input, i.e., by the position of the mixing valve v_m, which is limited to the range [0 1]. The temperature of the mixed glycol (T_{jin}) is not measured, but it can be estimated using the temperature of the input glycol, the jacket glycol temperature and the position of the valve. The time constants of the on/off valves and the mixing valves were neglected in this case. The mathematical model of the semi-batch reactor is defined by the differential Eqs. (37), (38), (40)–(42) and the algebraic Eqs. (39), (43).

$$m_j c_j \frac{dT_j}{dt} = v_m \Phi c_j T_{in} + (1-v_m)\Phi c_j T_j - \Phi c_j T_j - hS(T_j - T) - h_0 S_0 (T_j - T_0) \tag{37}$$

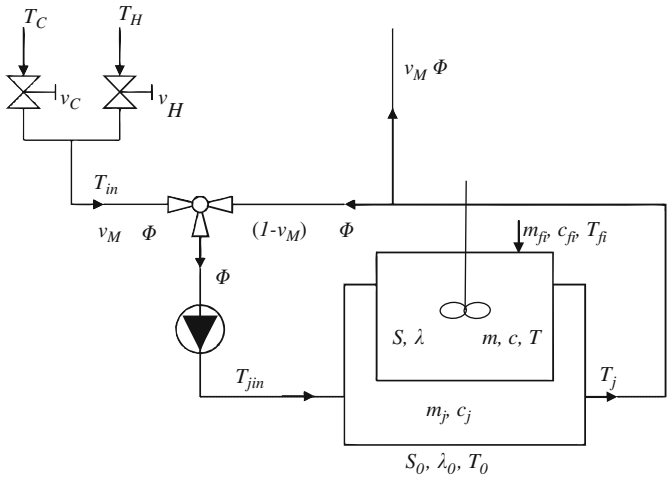

Fig. 1. The scheme of a semi-batch reactor process.

$$mc\frac{dT}{dt} = hS(T_j - (T + \Delta T\delta(t-T_i))) \tag{38}$$

$$\Delta T = \frac{m_{fi}c_{fi}(T_{fi}-T)}{mc + m_{fi}c_{fi}} \tag{39}$$

where m_j stands for the mass of the glycol in the jacket, c_j is the heat capacity of the glycol in the pipes, Φ is the mass flow in the pipes of the jacket, h is the heat-transfer coefficient from the jacket to the reactor's core. S represents the conduction area. The heat-transfer coefficient from the jacket to the surroundings is h_0 and S_0 represents the conduction area. The temperature of the surrounding is T_0. During the procedure more ingredients are added. This causes a change in the mass (m) of ingredients in the reactor, the heat capacity (c), the conduction area (S) and the temperature (T). These changes are much faster than the dynamics of the process; therefore, they can be modelled as discontinuous jumps. The jumps, which occur at time instants t_i are Dirac impulses $\delta(t-t_i)$. The discontinuous phenomenon of the system state (T) is modelled as follows from Eq. (38). The state jump (ΔT) is defined in Eq. (39), where m_{fi} stands for the mass of the ingredient added to the reactor at the time instant t_i, c_{fi} defines the heat capacity of the ingredient that is added and T_{fi} stands for the temperature of the ingredient at the time of loading. The variation of the mass inside the reactor is given in Eq. (40). Eq. (41) denotes the change in the average heat capacity of the mixture inside the reactor, where $\Delta c_i = (c_{fi}-c)m_{fi}/(m+m_{fi})$.

$$\frac{dm}{dt} = m_{fi}\delta(t-t_i) \tag{40}$$

$$\frac{dc}{dt} = \Delta c_i \delta(t-t_i) \tag{41}$$

The time-varying profile of the conduction surface, S, is given in Eq. (42):

$$\frac{dS}{dt} = S_{fi}\delta(t-t_i) \tag{42}$$

where S_{fi} stands for the change of the conduction surface at time t_i, due to the added ingredient. This is roughly modelled as $S_{fi} = S(m_{fi}/m)$.

The temperature in the reactor is controlled indirectly by the input jacket temperature (T_{jin}), which is now called the indirect control variable. T_{jin} depends on the position of the mixing valve and on the temperature of the fresh glycol Eq. (43):

$$T_{jin} = v_m T_{in} + (1-v_m)T_j \tag{43}$$

When using glycol for heating and cooling the reactor core, a nonlinear process characteristic is obtained. The heat-transfer

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coefficient from the jacket to the reactor's core is a nonlinear function of the temperature of the glycol in the jacket. Also, the heat capacity of the glycol in the jacket changes with its temperature. The approximate characteristic of the heat-transfer coefficient is shown in Fig. 2 (top graph), where the heat capacity is also shown (bottom graph).

4.2. The control of the process

The control of this process will be based on the indirect control variable (T_{jin}). This will then be transformed into the positions of the control elements, i.e., the position of the mixing valve (v_m) and the positions of the discrete valves (v_C and v_H). In order to do that a certain logic was introduced. First, if the reference is greater than the surrounding temperature T_0, which is kept constant then the switching is done as

$$\begin{aligned}&\text{if } e(k) < -\delta_d \text{ then } T_{in}(k) = T_C \ (v_C = 1, v_H = 0)\\ &\text{elseif } e(k) > -\delta_u \text{ then } T_{in}(k) = T_H \ (v_C = 0, v_H = 1)\\ &\text{else } T_{in}(k) = T_{in}(k-1),\end{aligned} \tag{44}$$

where δ_d and δ_u are switching thresholds and $e(k)$ is the control error.

If the reference is below or the same as the surrounding temperature the switching is done as:

$$\begin{aligned}&\text{if } e(k) > \delta_d \text{ then } T_{in}(k) = T_H \ (v_c = 0, v_h = 1)\\ &\text{elseif } e(k) < \delta_u \text{ then } T_{in}(k) = T_C \ (v_c = 1, v_h = 0)\\ &\text{else } T_{in}(k) = T_{in}(k-1).\end{aligned} \tag{45}$$

The logic behind this is that when the reference is above the temperature of the surroundings, the cooling in the vicinity of the reference can be done by cooling through the heat transfer from the jacket to the surroundings. But hot water is needed for the heating, and therefore the hot-water valve is kept open. And vice versa applies when the reference is below the surrounding temperature. Also, the saturation is added to the input of the process, so that the absolute difference between the jacket temperature and the reactor temperature is less than 60°.

Because of the nonlinear characteristic of the process (Figs. 3 and 4), the fuzzy predictive functional controller, described in Section 3, will be used. The Takagi–Sugeno fuzzy model can adequately model the nonlinear characteristic of the plant. Using 16 clusters and the first-order model a similar static characteristic to that of the plant is obtained. The comparison between the model's and the plant's characteristic is shown in Figs. 3 and 4, where the gain and the time constant are compared at different working points.

When new ingredients are added to the reactor, the dynamics of the reactor changes. To follow these changes and to properly adjust the controller's parameters, the recursive fuzzy identification described in Section 2 was used.

The controller has a set of parameters to be tuned. First, there are the parameters for the FPFC (a_r and H) and then there are also the parameters that affect the identification (forgetting factors, overlapping factor, fuzziness number of clusters, etc.). Although this is an adaptive controller, some prior knowledge of the process is needed to set the parameters.

The time horizon (H) of the FPFC is usually chosen as

$$1 \le H \le \frac{T_r}{2T_s} \tag{46}$$

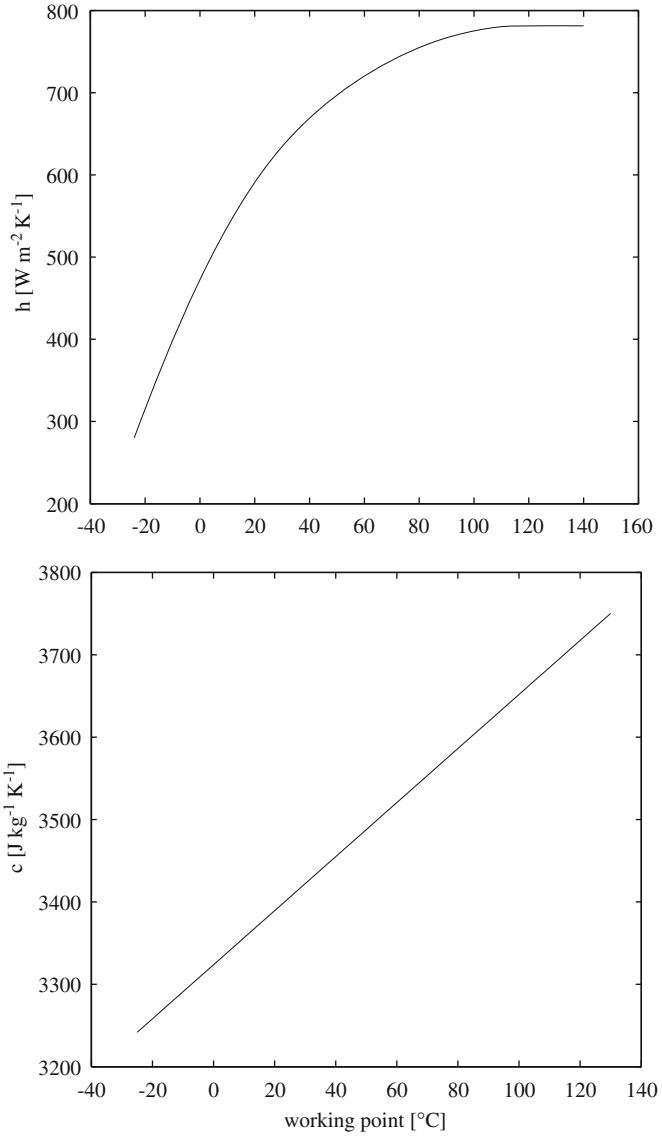

Fig. 2. The characteristic of the heat-transfer coefficient (top graph) and the glycol heat capacity (bottom graph).

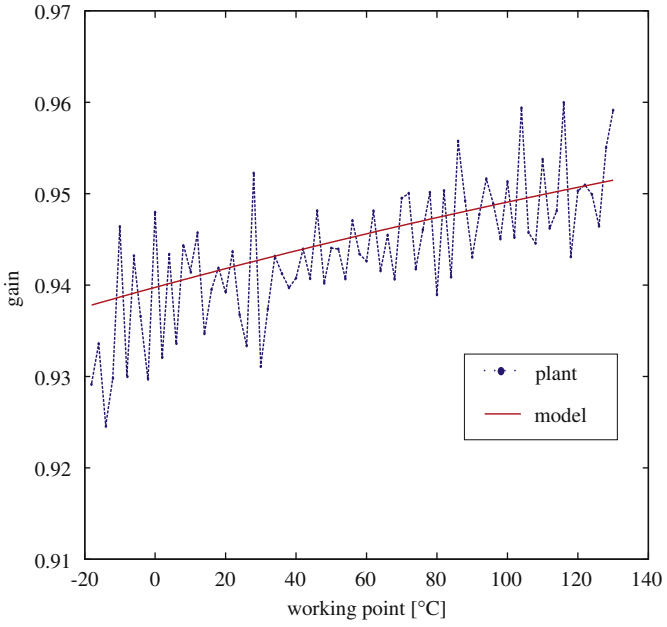

Fig. 3. Gain of the process (line with dots) and gain of the model (solid line).

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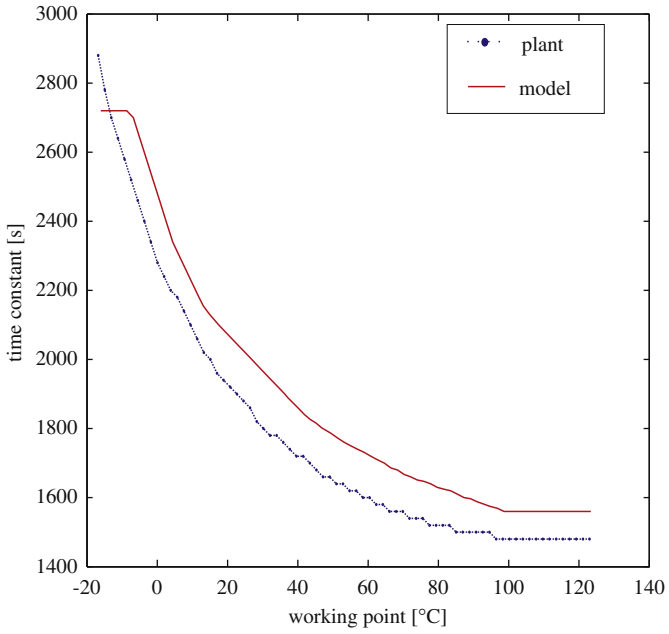

Fig. 4. The time constant of the process (line with dots) and the model (solid line).

where T_s is the sampling time and T_r is the time constant of the reference model (Škrjanc & Matko, 2000). The reference-model time constant is usually chosen based on the time constant of the process. In this case the reference-model (T_r) was chosen to be five time faster than the process.

The forgetting factors affect the speed of the adaptation and the smoothness of the estimates. For time-varying processes, the factors must be lower than one to ensure the forgetting. However, by lowering the forgetting factors the fluctuations of the estimates will increase. The values that give a good result are: $\lambda_r \approx \gamma_v \approx \gamma_c \approx 0.98-1$. If the changes in the process can be detected the forgetting factors can also be set to one, and when a change is detected the variance matrices can be reset (Åstrom & Wittenmark, 1995). A rule of thumb that indicates how many past samples will affect the model is

$$N = 2/(\lambda-1) \tag{47}$$

where λ is the forgetting factor and N is the number of samples that affect the model.

The fuzziness factors define the smoothness of the nonlinearity approximation. The higher they are the smoother the nonlinearity obtained, but too high values cause a larger identification error. The values that give a good result are: $\eta_m =$ from 0.25 to 1 and $\eta = 2$. The numbers of clusters are set based on a prior analysis of the input–output space.

5. Simulation

The adaptive fuzzy predictive functional control algorithm was tested on the described process using a simulation. The study was meant to show the potential of the proposed approach for further use on real nonlinear processes with time-varying nonlinearities and dynamics.

The process was simulated using the previously described equations. The parameters were as follows: the initial mass in the reactor core $m=600\,\mathrm{kg}$, the conduction area $S=2\,\mathrm{m}^2$, the heat capacity $c=4200\,\mathrm{J\,kg^{-1}\,K^{-1}}$ and the mass in the jacket $m_j=200\,\mathrm{kg}$. The heat capacity of the glycol and the heat-transfer coefficient from the jacket to the core were taken as shown in Fig. 2. The heat-transfer coefficient from the jacket to the surroundings $h_0=84\,\mathrm{W\,m^{-2}\,K^{-1}}$ and the conduction area $S_0=4\,\mathrm{m}^2$. The initial temperature was the same as the temperature of the surroundings $T_j(0)=T(0)=T_{in}(0)=T_0=17\,^\circ\mathrm{C}$. To make the simulation more real some noise, estimated from the real plant data, was added to the process. The input Gaussian noise of variance 0.03 was added to the reactor and jacket temperatures. The output noise was Gaussian of variance 0.08. This caused the output of the process (T) to vary for approximately $\pm 0.4^\circ$ peak to peak. In the simulated experiment the reactor is fed with an additional ingredient ($T_{fi}=17\,^\circ\mathrm{C}$, $m_{fi}=1000\,\mathrm{kg}$, $c_{fi}=4000\,\mathrm{J\,kg^{-1}\,K^{-1}}$).

For the identification algorithm and the controller the following settings were chosen: the identification regression vectors for the least squares were $\psi_i^T(k+1)=\beta_i(k)[1T(k-1)\ T_{in}(k-1)], i=1,\ldots,c$, the data vector was $x(k)=[T(k-1)\ T_{in}(k-1)T(k)]$, and the sampling time was $T_s=20\,\mathrm{s}$. The initial covariance matrices for the least squares were set to $P_i=100I$, $i=1,\ldots,c$, and the forgetting factor was set to $\lambda_r=1$. The initial values for the sub-models' parameters were set to zero. Also, a dead zone for the least squares was introduced to prevent *bursting*

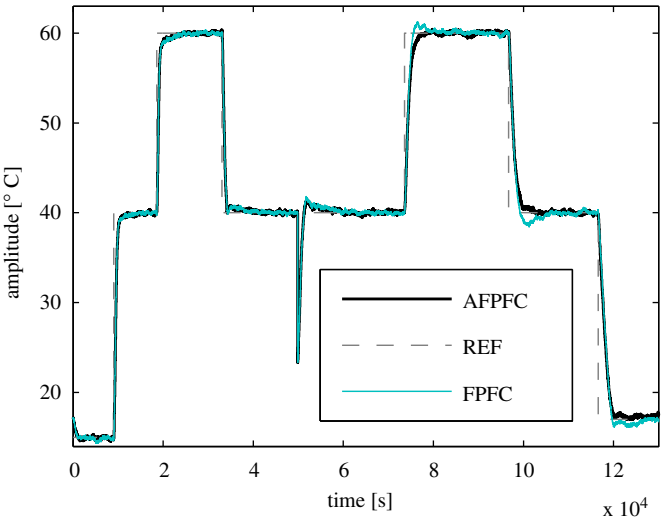

Fig. 5. Control of a semi-batch reactor.

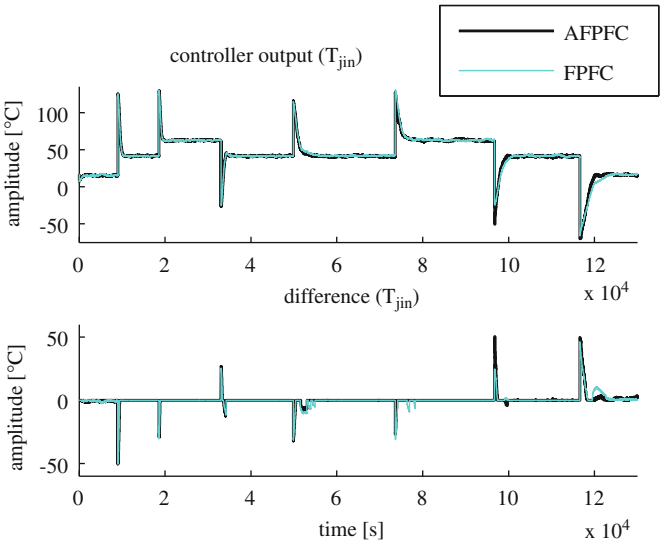

Fig. 6. Controller output (top graph) and difference between the real input to the process and the controller output (bottom graph).

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of the covariance matrix when there is little excitation. The constant of the dead zone was set to 0.01. The initial fuzzy covariance matrices were set to $F_i=1.5I$, $i=1,\ldots,c$ and the initial value of $s_i=1$, $i=1,\ldots,c$. The forgetting factors for the fuzzy covariance matrices and s_i were set to $\gamma_c=1$ and $\gamma_v=1$. The fuzziness factors were set as $\eta=2$ and the overlapping factor $\eta_m=0.3$.

Because of the nature of the process, the disturbance was detectable (the addition of ingredients). When the reference signal is constant and the output T changes significantly a disturbance is detected. At that time the covariance matrices P_i are reset to their initial values and the past membership degrees (s_i) and covariance matrices F_i of the fuzzy identification method are also reset by setting $\gamma_v=\gamma_v=0.0001$.

The controller parameters depend on the current model parameters. The relation between the model parameter a_m and the reference model parameter a_r was set as $a_r=\tilde{a}_m^5$, and the horizon was set as $H=round(-0.5(\log(a_r))^{-1})$. The switching threshold δ_d was 1 °C and δ_u was 0.5 °C.

In the experiment, first the fuzzy model of the plant was identified using perturbation signals at different working points. This model was then used as the initial model in the FPFC algorithm. Using the recursive fuzzy identification method the fuzzy model was updated during the regulation of the process.

In order to test the effectiveness of the adaptive FPFC, the method was compared to the standard non-adaptive FPFC. Fig. 5 shows the output of the process during the experiment. The output of the controller (the required value of the T_{jin}) is shown in Fig. 6 in the top graph, and in the middle graph the difference between the real and the required T_{jin} is presented. Fig. 7 shows the positions of the mixing valve (bottom graph) and the hot-water valve (top graph), from which the switchings between the hot and cold water can be seen.

Because the initial fuzzy model approximates the process very well, there is no major difference between regulation with the

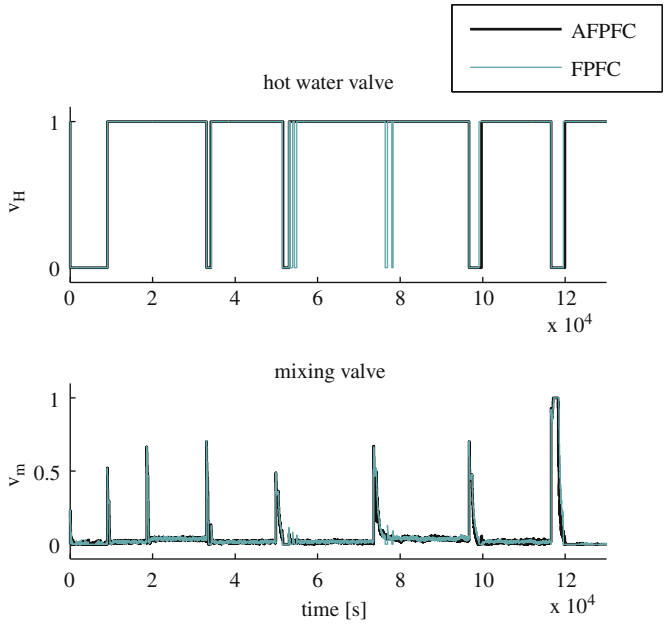

Fig. 7. The position of the hot-water valve (top graph) and the position of the mixing valve (bottom graph).

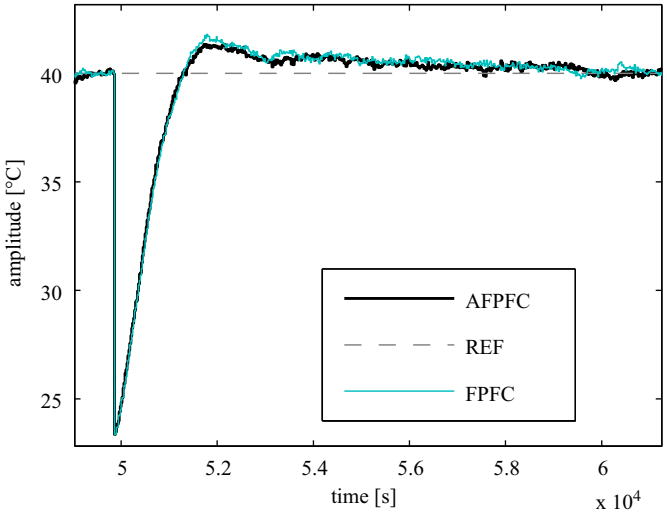

Fig. 8. Close up of addition interval.

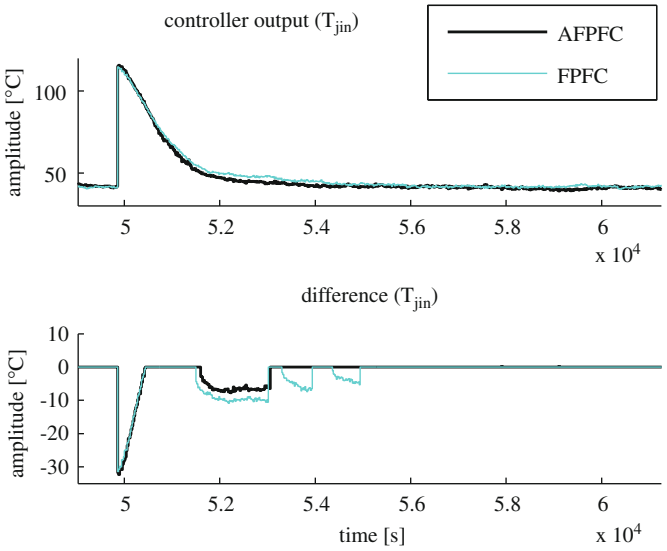

Fig. 9. Comparison of controller output (top graph) and difference between the real input to the process and the controller output (bottom graph) for the addition interval.

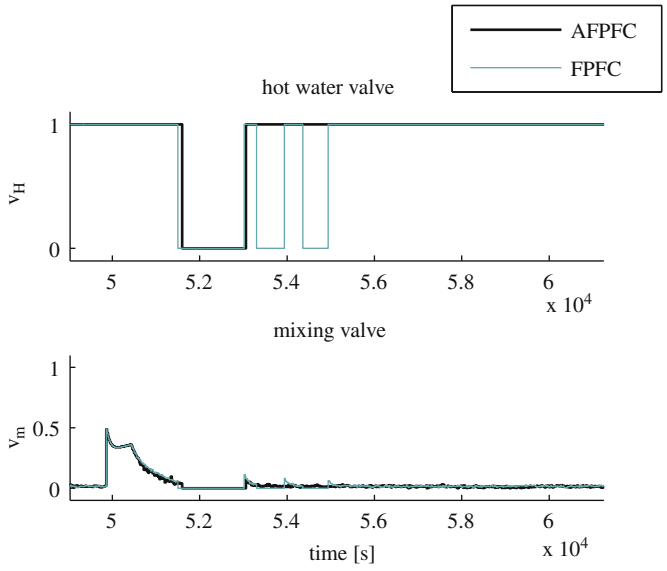

Fig. 10. Comparison of hot-water valve position (top graph) and mixing valve position (bottom graph) for the addition interval.

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adaptive and the non-adaptive FPFC. But when a disturbance occurs, in the form of adding new ingredients to the reactor, the process dynamic changes. Now the response of the process controlled with non-adaptive FPFC has an over-shoot. With the adaptive FPFC control the model is adapted to new process dynamics; therefore, the response has no over-shoot.

A detailed view of the disturbance rejection interval is shown in Figs. 8–10. In Fig. 8 the output of the reactor is presented. Fig. 9 shows the controller output (top graph) and the difference between the real input to the process and the controller output (bottom graph). Fig. 10 shows the position of the hot water-valve (top graph) and the position of the mixing valve (bottom graph). It can be seen that the speed of the disturbance rejection is about the same in both cases, but the overshoot is a bit smaller with the adaptive control. There is also less switching between the hot and cold water with the adaptive FPFC.

The interval after the addition is shown in Figs. 11–13. The adaptive FPFC adapts the model to a new dynamic; therefore, there is no overshoot, which is very important for controlling this kind of process. The overshoot affects the quality of the product in the reactor. There is also less switching between the hot and cold water.

The *SSE* values confirm the visual observations. The values were calculated as an average over 10 experiments. The *SSE* between the reference and the process output for the whole experiment is a little better with the adaptive FPFC (7.69e4) than with the non-adaptive FPFC (7.82e4). Also, switching between the hot and cold water is better with the adaptive FPFC. The adaptive FPFC has 10, and the non-adaptive FPFC has 18, switchings between the hot and cold water. And probably the most important value, which tells us the accuracy of the model is the *SSE* between the prediction (the internal model output) and the real output of the process. This was noticeably better with the adaptive control. The *SSE* between the model and process output, from the time when the disturbance was regulated until the end of the experiment, was 1.47e3 with the adaptive control and 6.68e4 with the non-adaptive control. The model predictions for the adaptive FPFC are shown in Fig. 14 and for the non-adaptive in Fig. 15.

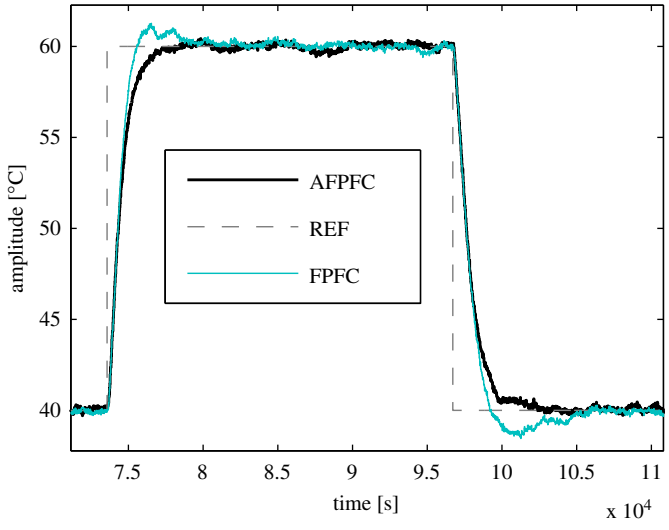

Fig. 11. Close up of interval after the addition.

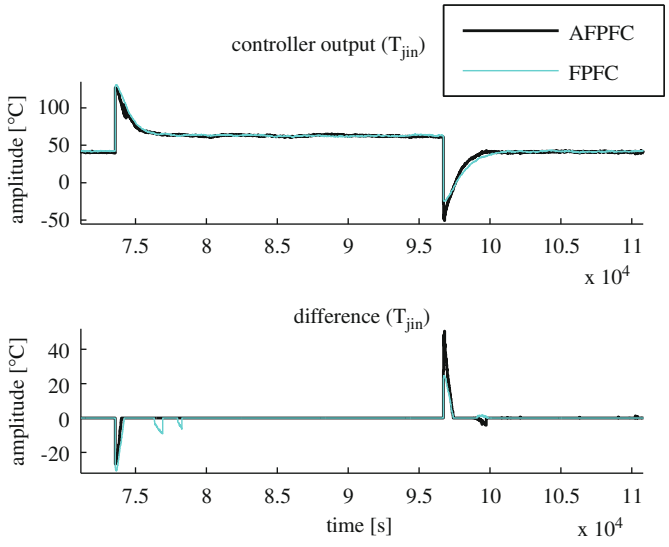

Fig. 12. Comparison of controller outputs (top graph) and the difference between the real input to the process and the controller output (bottom graph) for the interval after the addition.

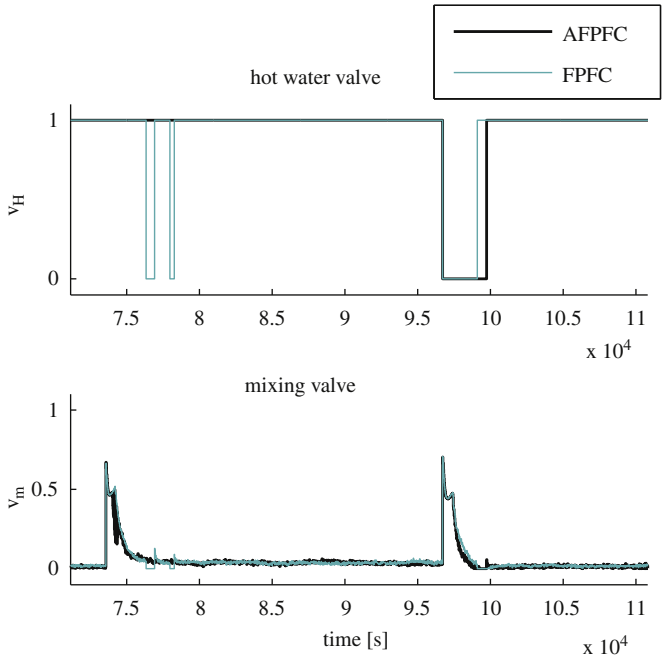

Fig. 13. Comparison of the hot-water valve position (top graph) and the mixing valve position (bottom graph) for the interval after the addition.

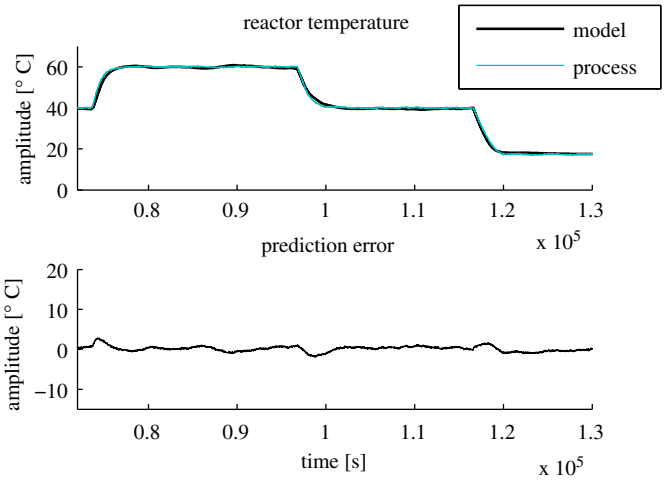

Fig. 14. The model prediction and the process output for adaptive FPFC (top graph) and the prediction error (bottom graph).

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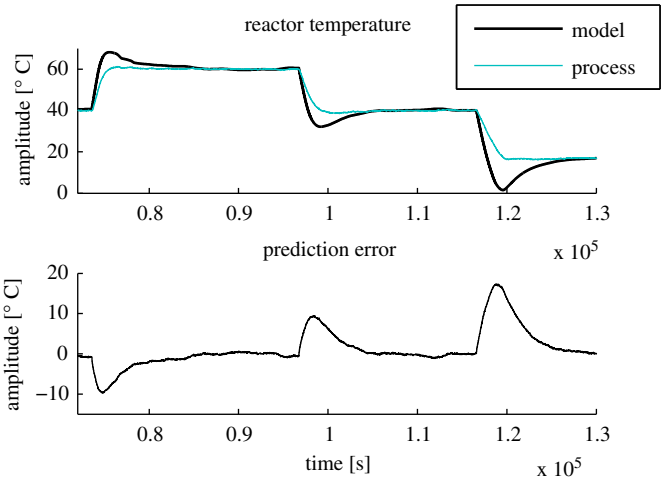

Fig. 15. The model prediction and the process output for non-adaptive FPFC (top graph) and the prediction error (bottom graph).

6. Conclusion

This simulation study was made to elaborate the possible use of a developed recursive fuzzy *c*-means clustering algorithm in connection with fuzzy predictive functional control to construct the adaptive FPFC. The adaptive version of the FPFC was compared to the standard FPFC for a semi-batch reactor process. This is a nonlinear process with time-varying parameters.

The results indicate that the adaptive fuzzy predictive functional control can cope better with parameter changes than the fuzzy predictive functional control. The disturbance response as well as the response to a step reference change has less overshoot. This ensures better quality of the product. There is also less switching between the hot and cold water, which is important for the longevity of the actuators.

The adaptive fuzzy predictive functional control can be used to improve the control of the time-varying nonlinear processes, especially those where the instants of the changes can be detected and where the parameters (nonlinearity) are constant (in the sense of time dependency) over a long period of time and change abruptly only occasionally.

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